Developing a Tool for Automatic Stability Verification of Hybrid Systems

Henning Burchardt, Jens Oehlerking, Oliver Theel
University of Oldenburg
26111 Oldenburg, Germany
{henning.burchardt, jens.oehlerking, oliver.theel}@informatik.uni-oldenburg.de

The transregional research collaboration AVACS (Automatic Verification of Complex Systems) is a project spanning the three German universities of Freiburg, Oldenburg and Saarbrücken, as well as the Max Planck Institute for Computer Science in Saarbrücken. One main research area is the verification of so-called *hybrid systems*, that is feedback control systems that combine continuous-time dynamics with discrete state jumps. Hybrid systems can be visualized as feedback control system containing a controller with multiple modes of operation, each with different continuous-time dynamics. The switching logic between the modes is defined as a function of the current mode of operation and the continuous system state.

In the context of AVACS, we are developing a tool for automated stability verification of hybrid systems. The tool will be able to deal with a variety of hybrid systems and a variety of stability notions.

To achieve this, we employ a modular approach, allowing us to include as many verification algorithms as possible. Most methods for proving stability of a hybrid system only work for a certain class of systems or a particular stability notion. Classes include piecewise affine systems, linear hybrid automata, systems with polynomial dynamics, et cetera. Stability notions include Lyapunov stability, asymptotic stability, exponential stability or convergence to a region. To identify suitable algorithms for a given pair of hybrid system and proof obligation, we use a classification tool that takes a model of a hybrid system as input and analyzes the dynamics. This gives us the class of systems this particular hybrid system belongs to and, together with the proof obligation, allows us to select and run a suitable algorithm.

Another advantage of this modular design is that the tool is easily extendible through new algorithms, so that the class of systems covered can gradually increase.

The modelling language used for hybrid systems is based on first-order logic. Constraints for the continuous dynamics (*flows*) and discrete updated (*jumps*) are divided into guards and invariants. For flows, the guard describes the mode corresponding to a particular dynamics and the invariant contains the corresponding differential equation. For jumps, the guard represents the so-called *switch set* and the invariant gives the new mode of the system, as well as a possible update of the continuous variables.

The verification algorithms currently in development include abstraction-refinement based methods suitable for proving convergence to a region, as well as methods for asymptotic stability. The latter are outlined on the poster, utilizing the well-known concept of Lyapunov functions. A Lyapunov function can be seen as an energy function of the system, with the energy decreasing over time and converging to zero. The existence of a Lyapunov function proves asymptotic stability of the system. We employ *linear matrix inequality (LMI)* based methods as proposed by Johansson and Rantzer [1] and Pettersson [2] to generate a Lyapunov function for a system.

The method consists of the three steps. First one needs to partition the state space of the system into several regions. For each region, we will try to find a separate function with Lyapunov-like properties. Secondly, the problem of finding such functions is formulated as a LMI problem, using knowledge about possible transitions between the regions. The solution of this LMI problem will be the desired family of functions, acting as a a certificate of stability. Thirdly, this LMI problem is solved using convex optimization tools.

As of now, the tool is still in development, we hope to present some results in the near future.

References

[1] Mikael Johansson and Anders Rantzer. Computation of piecewise quadratic Lyapunov functions for hybrid systems. *IEEE Transactions on Automatic Control*, 43, 1998.

[2] Stefan Pettersson. *Analysis and Design of Hybrid Systems*. PhD thesis, Chalmers University of Technology, Gothenburg, 1999.